Users Caching Two Files: An Improved Achievable Rate

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What is Caching?

- Two-phase communication between a server and several clients
- Placement: each client stores a function of the files of size \( M \) inside its local cache!
- Delivery: each client requests a file. The server broadcasts an update message of size \( R \).
- Find \( R(M) \), the minimum update rate such that each user can decode its desired file!

Maddah-Ali–Niesen Coded Caching Strategy

- \( N \) files, \( K \) users, cache size per user \( M = \frac{K}{N} \).
- Each user caches \( \frac{1}{N} \) bits of each file in a symmetric way.
- Delivery is based on linear index coding with side information.

Achievable rate is

\[
R(M) = K \times (1 - M/N) \ \cdot \ \ \min \left( \frac{1}{1 + KM/N} \right) \ \cdot \ \frac{N}{K}
\]

- Performs poorly when \( K < N \) and \( M < 1 \).

Proposed Caching Strategy for Two Files; Placement

- Based on Simultaneous coded placement and coded delivery.
- Toy example: \( K = 4, N = 2, M = 1/2 \). Placement phase:

<table>
<thead>
<tr>
<th></th>
<th>A13</th>
<th>A12</th>
<th>A14</th>
<th>B12</th>
<th>B13</th>
<th>B14</th>
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</thead>
<tbody>
<tr>
<td>Z1</td>
<td>A13</td>
<td>A12</td>
<td>A14</td>
<td>B12</td>
<td>B13</td>
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<td>Z2</td>
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<tr>
<td>Z4</td>
<td>A13</td>
<td>A12</td>
<td>A14</td>
<td>B12</td>
<td>B13</td>
<td>B14</td>
</tr>
</tbody>
</table>

Proposed Caching Strategy for Two Files; Delivery

- Assume \( d_{(1,2,3,4)} = \{ A, A, B, B \} \).
- Transmit \( A_{13}, A_{23}, A_{14}, A_{34}, B_{12}, B_{23} \).
- Transmit \( B_{13} \oplus B_{14} \) and \( B_{23} \oplus B_{24} \).

General Algorithm and Achievable Rate

Placement:

\[ T \subseteq \{ 1, \ldots, K \}, \text{s.t.} \ |T| = m, \forall k \in [1 : K], \]

- If \( k \in T \), store \( A_T + B_T \) in \( Z_k \).

Delivery: suppose \( d_{(1,...,L)} = \{ A, \ldots, A \} \) and \( d_{(L+1,...,K)} = \{ B, \ldots, B \} \).

- \( \forall T \in \{1, \ldots, L\} \), \( S \subseteq \{ L + 1, \ldots, K \} \) s.t. \( |S| + |T| = m \)
- Suppose \( |S| \geq j \), transmit \( A_{T \cup S} \); Else, transmit \( B_{T \cup S} \).

- \( \forall T \in \{L + 1, \ldots, K\} \), \( S \subseteq \{ L + 2, \ldots, K \} \) s.t. \( |S| + |T| = m, |S| < j \)
- \( \forall T \in \{L + 1, \ldots, K\} \), \( S \subseteq \{ L + 2, \ldots, K \} \) s.t. \( |S| + |T| = m, |S| \geq j \)

Achievable Rate: \( R_K(M) = \max_{0 < L < K} R_K(M, L) \) where

\[
R_K(M, L) = 1 + \sum_{i=0}^{L-1} \frac{\binom{K-1}{i} \binom{L-1}{i}}{\binom{K}{i}} + \sum_{i=K}^{L-1} \frac{\binom{K-1}{i} \binom{L-1}{i}}{\binom{K}{i}}
\]

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