Finding the dictionary $A_{m \times n}$ such that $y_j = Ax_j$, from a set of $K$ observations $y_1, \ldots, y_K \in \mathbb{R}^m$ is called dictionary learning.

**Applications:**

- Inpainting image (removing the text from the image). Taken from [2]
- Visualizing voting pattern in Switzerland. Taken from [1]

The problem is only solvable with additional assumptions.

**Sparse component analysis (SCA):**

- $x_1$ is sparse.
- Recover $A$ using a cost function that promotes the sparsity. For example:

$$\hat{A}_{y_i} = \arg\min_B \frac{1}{2K} \sum_{j=1}^{K} \|y_j - Bx_j\|_2^2$$

subject to $\|x_j\|_1 \leq \lambda$.

**Independent component analysis (ICA):**

- The entries of $x_1$ are independent.
- The dictionary $A$ is found by using a cost function that measures independence of entries of $x$.

Our assumption:

$x_1$ follow a symmetric $\alpha$ stable distribution (S\alpha S).

Why model the sparse signal as an S\alpha S signal?

- They model a large class of sparse signals.

**Sparse representation for color image restoration:**

No need to recover $x_1, \ldots, x_K$.

Hyper-parameter free.

$x_1$ does need to be exactly 0, so more robust to noise.

**Results:**

Preliminary results from the experiments on synthetic data:

$\hat{x}_1, \hat{x}_2 \sim S\alpha S, A \sim N(0, I_{m \times m})$.

**Benchmarks:**

- $\ell_1/\ell_2$: Minimizing $\sum_{i=1}^{K} \|x_i\|_1$ conditioned on $\|y_1 - \hat{A}x_1\|_2 \leq \lambda$.
- $\ell_1 + \ell_2$: Minimizing $\frac{1}{2} \sum_{i=1}^{K} \|x_i - Bx_i\|_2^2 + \lambda_1 \|x_1\|_1 + \lambda_2 \|x_1\|_2^2$.

$\lambda$ controls the sparsity.

![Image](image.png)

**Figure 6:** Impact of $\alpha$ on the success rate of the algorithms. The interesting range for many applications including image processing is roughly $\alpha \in [1, 1.5]$.

**Filling missing pixels:**

![Image](image.png)

**Figure 7:** Filling missing pixels with a dictionary trained over face dataset.

**References:**


Mining democracy.


Online learning for matrix factorization and sparse coding.


Sparse representation for color image restoration.