Optimal Software Patching Plan for PMUs
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Power Grid Observability Rules

Rule 1: A bus is observable if it is a PMU bus.

Rule 2: A bus is observable if it is adjacent to at least one PMU bus.

Rule 3: For any set of buses made of an injection bus and all its adjacent buses, if we know that all but one bus in the set are observable, then all buses in the set are observable.

Rule 4: If a flow measurement (active and reactive) of a branch is known if and only if one of its terminal buses is observable, the other terminal bus is observable.

PMU Patching Problem

Given a power grid of n buses B=\{1,2,3,...,n\} that has a set of m deployed PMUs P=\{1,2,3,...,m\} with enough redundancy, how do we find a plan that rolls out a software patch to all PMUs in as few rounds as possible without losing full grid observability?

Can be modeled as a sensor patching problem (SPP).

Sensor Patching Problem (SPP)

Given a set of n sites to be monitored S=\{1,2,3,...,n\}, a set of m sensors P=\{1,2,3,...,m\} that monitor the sites and \(\Gamma_S: S \rightarrow P^m\); i.e., \(\Gamma_S\) is the set of sensors monitoring site \(s\), find a partition \(\{c_1,c_2,...,c_k\}\) of \(S\) such that \(k\) is minimum and:

\[|\Gamma_S|_{c_i} \geq 1, \forall s \in S, \text{ and } i=1,2,...,k? \quad (1)\]

SPP Decision Problem:
Instance: B, P, \(\kappa_2\)
Question: Is there a partitioning of \(S\) into at most \(\kappa_2\) disjoint subsets \(\{c_1,c_2,...,c_k\}\), such that (1) is satisfied?

Mapping PMU Patching Problem to SPP

Two-Step Grid Topology Transformation

Step 1: Merging an injection bus with a neighbor
Step 2: Replace terminal buses of branches with flow measurements by another bus

Obtain sets \(B'=\{1,2,...,\ast\}\), \(\Gamma(b')\) and \(P\) from transformed topology

PMU patching problem of the transformed topology mapped to an SPP by mapping \(B'\) to \(S\), \(P\) to \(P\), and \(\Gamma(b')\) to \(\Gamma(s)\)

NP-completeness of SPP

SPP’s NP-completeness can be proved by reducing an instance of HCP(V',E',k) where \(\varepsilon \subseteq 2\), \(\varepsilon \subseteq E\) to an instance SPP(P',S',\(\Gamma\), \(\kappa\)) in polynomial time using the following transformation:

\(P' \subseteq V', S' \subseteq E', k \subseteq \Gamma, \Gamma \subseteq H_{d1}\) such that \(\varepsilon \in \Gamma, \varepsilon \in E\).

Since HCP is NP-complete, thus SPP is also NP-complete.